Separating Productivity and Quality Effects from Relative Prices

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Abstract

Based on a general equilibrium model, we derive a relative price function which can be decomposed into productivity and quality effects. We then develop a method for inferring relative quality changes and apply that to US services versus US goods using NIPA data from 1946-2006. First, relative quality was decreasing after 1946 and has been increasing since the 1970s. Second, relative productivity and quality are negatively correlated, suggesting an endogenous link between the two. Third, productivity changes alone cannot fully explain the evolution of the services-goods relative price. This suggests that ignoring quality variations when explaining relative prices or exchange rates can lead to incorrect conclusions.

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1 Introduction

Relative prices are understood as conversion rates between commodity bundles of different countries or sectors. In standard growth and business-cycle models, relative prices vary due only to productivity shocks. *Ceteris paribus*, relative price and relative quantity, which is driven by relative productivity, should have a negative correlation. That is as we consume more of some product, marginal utility and hence relative price of that product will decrease. This negative correlation does not fully hold for the case of US services versus US goods

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from 1946-2006 (Figure 1): while services-goods relative quantity fluctuates, the relative price steadily increases over time, suggesting that the standard models may miss something important in the economy.

Potentially, there are other important sources of dynamics besides productivity. Among those is quality innovation–changes in utility level given the same consumption quantity. In the current study, we allow the coexistence of productivity shock and quality innovation, which are both exogenous. Given this coexistence, the study addresses three closely related questions: (i) with data on relative prices and potentially other variables, how can we separate the effects of productivity shock and quality innovation? (ii) given the separation, how are they individually and jointly characterized for the US, i.e. volatility, persistence, correlation and causation? and finally (iii) what are the implications of the separation exercise to a certain set of business-cycle and growth models?

The study shows that based on a simple general equilibrium model, we can separate productivity and quality. Specifically, we can infer relative quality changes using time series of relative prices and budget shares. In addition, with a Vector Auto Regressive (VAR) model, we can analyze the dynamic relationship between productivity and quality. The empirical results based on US data show that quality innovation plays an important role in variations of the services-goods relative price, and productivity shock alone cannot fully explain the behavior of this relative price. This implies that models with only productivity shocks may generate misleading results. Stockman & Tesar (1995) reported that the addition of a taste shock between tradeables and non-tradeables helps better explain some international stylized facts which are hard to arrive at with productivity shock alone.

There is a new and growing literature on inferring quality from price information. Klenow (2003) provided a detailed critical review on the efforts by different statistical agencies in separating quality improvements from price changes, and gave some practical suggestions. Bils & Klenow (2004) decomposed inflation in unit prices of 66 US durable consumer goods into quality and pure-price effects. Hummels & Klenow (2005) looked at many countries' detailed exports data and found that richer countries charge higher prices which result from better quality. Hallak (2006) retrieved quality from export unit prices at the sectoral level and confirmed the theoretical prediction that rich countries buy relatively more from countries of high quality goods. The major weakness of this literature is that quality is retrieved either with a focus on prices alone or with inadequate specifications of sectoral production.

In this empirical quality literature, the closest work to the current study is Hallak & Schott (2005). They used relative prices and sectoral trade balances to decompose export unit value into quality and non-quality components. Their argument is: given a common international price for some sector, a country has a positive trade balance for that sector if its quality is higher than that of the trading partner. This argument does not always hold. For example, in a simple world where quality levels are the same and the law of one price holds, we can see non-zero sectoral trade balances: subject to disproportional quantities of sectoral endowments, countries benefit from net selling some product and net purchasing another. Moreover, their argument is hard to be extended to the aggregate level: overall trade balances partly reflect intertemporal consumption smoothing, which is not related to quality.

Our study has an aspect similar to that of the huge literature on demand empirics: looking at implications of utility maximization. However, the objectives of our study and those of the literature on demand empirics are different. The empirical demand literature has two major lines: (i) parametric approach in different flexible forms, e.g. the path-breaking paper by Diewert (1971) and a good empirical comparison by Fisher et al. (2001); and (ii) nonparametric approach with the generalized axiom of revealed preferences, e.g. Varian (1982, 1983). Both of these lines are concerned with the consistency between preference axioms and aggregate data. Our focus is on how aggregate quality is changing over time. Under the hypothesis that quality does change, the tests in parametric and nonparametric approaches have some problems. First, in different flexible forms of utility or indirect utility, consumption quantities are the only objects that evolve over time and all parameters are fixed. If quality and hence marginal utility are evolving, the parameters in those specifications should change, i.e. each period has some preference structure which is consistent with data of that period only. Consequently, with intensity in parameters, different flexible forms are hard to be properly implemented with aggregate data to capture quality innovation. Second, if quality varies and therefore the set of commodities evolve over time, we cannot apply the test of generalized axiom of revealed preferences when preferences may be quite different between periods. For these reasons, we rely on the parsimony in parameters to track quality changes and choose the constant elasticity of substitution (CES) utility function.

We focus on relative productivity and quality at the aggregate level and have some contributions to the literature. First, separation of productivity shock and quality innovation is based on relative prices and budget shares. This means that the retrieval of quality innovation fully takes preference and technology into account. Second, measures for goodness of fit, which tell how much productivity shock and quality innovation explain relative price and budget share, are developed. Based on these measures, we also know how important the measurement errors are in a specific economic context. Third, via an application, we know the evolution of US services-goods relative quality from 1946-2006. Fourth, by imposing a VAR structure on relative productivity shock and quality innovation, we have some insights on their individual and joint properties, which will serve as moments for further studies.

The rest of the study is structured as follows. Section 2 lays out the basic environment and focuses on productivity and quality information possibly borne by relative price variations. Section 3 extends the basic model by using both relative price and budget share to deal with measurement problems. The main result is an inference procedure for retrieving the relative quality index. Section 4 applies the methods developed earlier to US services-goods data. From this empirical analysis, we learn about the evolution and importance of quality innovation in the US context. Finally, we close the study with some remarks.

2 An Endowment Economy

2.1 The Basic Model

We have an economy populated by a unit measure of identical agents. In each period, the agents are endowed with commodities a and b and they can freely trade those endowments to satisfy their need. A typical agent $i \in [0,1]$ solves the following static CES utility maximization problem at time t

$$\max_{\{a_{it},b_{it}\}} \left\{ (\alpha_t a_{it})^\theta + (\beta_t b_{it})^\theta \right\}^{1/\theta} \tag{1}$$

subject to the budget constraint

$$a_{it} + p_t b_{it} = e_{ait} + p_t e_{bit}, \tag{2}$$

where (a_{it}, b_{it}) are consumption quantities; (α_t, β_t) are positive quality indices of commodities a and b, respectively; (e_{ait}, e_{bit}) are endowment quantities; p_t is the relative price which denotes the amount of commodity a needed to trade for a unit of commodity b; and $\theta = 1 - 1/\sigma$, where σ is the constant elasticity of substitution (absolute value). In the literature, θ is called the *substitution parameter*. As the elasticity of substitution σ belongs to $[0, \infty)$, the substitution parameter θ lives in $(-\infty, 1]$. Essentially, we have a CES utility function in which effective consumption quantity is a product of quantity and quality. In this paper, changes in (α_t, β_t) are interpreted as quality innovation rather than taste shock. It is hard to interpret taste shock as a synchronized event happening to all agents, especially with a time length unit of one year or more. However, quality innovation can come from competition and imitation in production. With the restricted space of $\{\alpha_t, \beta_t, \theta\}$, marginal utilities are positive and decreasing. In addition, the utility function satisfies the Inada condition.

The CES specification in (1) covers a broad range of substitutability. Arrow et al. (1961) showed that: (i) CES is fixed-proportion Leontief ($\sigma = 0$) for $\theta = -\infty$; (ii) CES is inelastic ($0 < \sigma < 1$) for $\theta \in (-\infty, 0)$; (iii) CES becomes Cobb-Douglas ($\sigma = 1$) for $\theta = 0$; (iv) CES is elastic ($1 < \sigma < \infty$) for $\theta \in (0, 1)$; and CES has straight-line indifference curves ($\sigma = \infty$) for $\theta = 1$. In addition, the desired budget share for, without loss of generality, commodity a is a positive function of the coefficient α_t^θ.

On the technology side, total endowment quantities in any period t are

$$E_{at} = A_t \tag{3}$$

$$E_{bt} = B_t, \tag{4}$$

where the quantity ratio B_t/A_t follows some stochastic process. As the agents equally share the endowments, $e_{ait} = A_t$ and $e_{bit} = B_t$ for every i and t. Besides quantity, the quality ratio β_t/α_t also evolves stochastically. In this study, A_t and B_t are mentioned as productivity.

Let $\omega_t = \{A_t, B_t, \alpha_t, \beta_t\}$ be the information set. The timing in period t is: (i) at the beginning of the period, quality indices and quantity shocks in ω_t are fully observed by all agents; (ii) based upon this information set, the agents figure out their consumption plans; (iii) and then they trade with competitive terms in the markets.

Definition 2.1. *A competitive equilibrium consists of the quantity and price functions* $\{a_{it}(\omega_t), b_{it}(\omega_t), p_t(\omega_t)\}_{i\in[0,1],t\geq 0}$ *which satisfy the following conditions in any period* t*:*

(i) Given some information set ω_t *and price* p_t*,* $\forall i$*,* $\{a_{it}, b_{it}\}$ *maximize agent* i*'s utility in (1) subject to the budget constraint in (2);*

(ii) Given the information set ω_t *and the consumption plans in (i), price* p_t *clears the markets:*

$$\int_0^1 a_{it}di = A_t \tag{5}$$

$$\int_0^1 b_{it}di = B_t. \tag{6}$$

As all agents are identical, $a_{it} = A_t$ and $b_{it} = B_t$ $\forall i$. After some manipulations (Appendix A.1), we derive the equilibrium relative price as

$$p_t = \left(\frac{\beta_t}{\alpha_t}\right)^{\theta} \left(\frac{B_t}{A_t}\right)^{\theta-1} \tag{7}$$

with the first-order derivatives

$$\frac{\partial p_t}{\partial (B_t/A_t)} = (\theta - 1)\frac{p_t}{B_t/A_t}, \tag{8}$$

$$\frac{\partial p_t}{\partial (\beta_t/\alpha_t)} = \theta\frac{p_t}{\beta_t/\alpha_t}. \tag{9}$$

There are some terminological notes. First, sector 2 has a *favorable productivity shock* if the ratio B_t/A_t is higher than that in the previous period. Second, sector 2 has a *favorable quality innovation* if the ratio β_t/α_t becomes higher. These notes also apply to sector 1 with respect to the ratios A_t/B_t and α_t/β_t. Third, *relative price* of a sector tells how many units of the other commodity needed to trade for one unit of this sector's commodity. We have the following results:

Proposition 2.2. *Ceteris paribus,*

(i) when a sector has a favorable productivity shock, its relative price depreciates if $\theta < 1$*, and stays the same if* $\theta = 1$*;*

(ii) when a sector has a favorable quality innovation, its relative price depreciates if $\theta < 0$*, remains unchanged if* $\theta = 0$*, and appreciates if* $\theta \in (0, 1]$*.*

Proof. These results can be directly inferred from (8) and (9). ∎

Table 1 summarizes the results in Proposition 2.2. We clearly see the qualitative effects of two sources of variations on the equilibrium relative price. It

Table 1: Partial effects of different variations on relative price

	$\theta < 0$	$\theta = 0$	$0 < \theta < 1$	$\theta = 1$
favorable productivity shock	$-$	$-$	$-$	0
favorable quality innovation	$-$	0	$+$	$+$

is interesting to note that when $0 < \theta < 1$, a favorable productivity shock and a favorable quality innovation have opposite effects on relative price. The following discussions will provide more intuition behind these results.

Partial Effects of Productivity Shock. When $\theta < 1$ or the utility function is strictly concave, an increase in the relative endowment of either commodity will eventually push down the marginal utility of that commodity relative to the other's, and hence the relative price decreases. When $\theta = 1$ or the commodities are linearly substitutable, marginal utility is constant given some quality indices, and relative price is not affected by productivity shock. Generally, relative productivity and relative price have a negative correlation.

Partial Effects of Quality Innovation. When $\theta < 0$, an increase in relative quality of either commodity will push down the relative price of that commodity, given some endowments. Interestingly, this result seems counter-intuitive at the face value. To see why the result is true, note that this is the case where products are hard to be substituted, which means the utility function is quite concave with respect to quantity and quality (Figure A.1). Thus, an increase in quality may shift up the utility function faster at low quantity than at high quantity, reducing marginal utility at each quantity level. The opposite effects apply for $\theta > 0$ (Figure A.2). That is an increase in quality may shift up the utility function faster at high quantity than at low quantity, increasing marginal utility at each quantity level. When $\theta = 0$, we have a Cobb-Douglas utility function where quality innovation does not affect relative marginal utility.

In addition to the equilibrium relative price p_t, we have the equilibrium budget share for commodity a (S_{at}) as

$$S_{at} = \frac{1}{1 + p_t \frac{B_t}{A_t}}, \text{ or} \tag{10}$$

$$S_{at} = \frac{1}{1 + \left(\frac{\beta_t}{\alpha_t}\right)^{\theta} \left(\frac{B_t}{A_t}\right)^{\theta}}. \tag{11}$$

It is noted for expositional simplicity, we use S_{at} rather than S_{bt}. From (7) and (11) we see that variations in productivity shock and quality innovation are manifested by both relative price and budget share. This result is coined *double manifestation*. Thus, if productivity shock, relative price, and budget share can be perfectly observed, there are two alternative ways to infer quality innovation, i.e. either with relative price or budget share. For example, given relative quantity and price data, we can estimate the substitution parameter with a regression, possibly with some instruments, as follows

$$\ln p_t = \lambda_0 + (\theta - 1)\ln(B/A)_t + \varepsilon_t. \tag{12}$$

and calculate the quality index as

$$(\beta/\alpha)_t = \left[p_t / (B/A)_t^{\theta-1}\right]^{1/\theta}. \tag{13}$$

If we have a world of two countries each endowed with one of the commodities a and b, and free trade takes place, the equilibrium terms of trade will also have the form in (7). Thus, even though the model is explicitly about a closed economy, its essentials can be extended to international contexts. However, those potential extensions will need many additional considerations, which are beyond the scope of this study.

2.2 VAR and a Dynamic Relationship

We may see different correlation patterns between productivity shock and quality innovation, as long as we already know both of the series, hereafter, $(B/A)_t$ and $(\beta/\alpha)_t$. We choose a simple VAR model to analyze the dynamic relationship between them. It is noted that the VAR model is not used to separate quality innovation. It is used to generate some moments of interest.

We are developing a simple procedure to learn about variance, persistence, causation, and correlation of productivity shock and quality innovation, which are assumed to follow a lag-1 VAR model. The quantity process is characterized by mean μ_p and standard deviation (STD) σ_p. The quality process has mean μ_q and STD σ_q. Quantity and quality have a correlation coefficient of φ and a corresponding covariance of $\sigma_{pq} = (\sigma_p \sigma_q)\varphi$. We construct two new random variables as deviation from mean

$$P_t = (B/A)_t - \mu_p \tag{14}$$

$$Q_t = (\beta/\alpha)_t - \mu_q. \tag{15}$$

By construction, $var(P_t) = \sigma_p^2$, $mean(P_t) = 0$, $var(Q_t) = \sigma_q^2$, $mean(Q_t) = 0$, and $covar(P_t, Q_t) = \sigma_{pq}$. The VAR model for (P_t, Q_t) is specified as

$$\begin{bmatrix} P_t \\ Q_t \end{bmatrix} = \begin{bmatrix} \lambda_{pp} & \lambda_{qp} \\ \lambda_{pq} & \lambda_{qq} \end{bmatrix} \begin{bmatrix} P_{t-1} \\ Q_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{qt} \end{bmatrix}, \tag{16}$$

where

$$\begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{qt} \end{bmatrix} \overset{i.i.d}{\sim} N(\mathbf{0}, \Sigma) \text{ and } \Sigma = \begin{bmatrix} \gamma_p^2 & \gamma_{pq} \\ \gamma_{pq} & \gamma_q^2 \end{bmatrix}. \tag{17}$$

For the sake of simulations, we need to specify Σ based upon characteristics of (P_t, Q_t). We have the following relationship (Appendix A.2)

$$\begin{bmatrix} \gamma_p^2 \\ \gamma_q^2 \end{bmatrix} = \begin{bmatrix} 1-\lambda_{pp}^2 & -\lambda_{qp}^2 \\ -\lambda_{pq}^2 & 1-\lambda_{qq}^2 \end{bmatrix} \begin{bmatrix} \sigma_p^2 \\ \sigma_q^2 \end{bmatrix} - \begin{bmatrix} 2\lambda_{pp}\lambda_{qp}\sigma_{pq} \\ 2\lambda_{pq}\lambda_{qq}\sigma_{pq} \end{bmatrix}, \tag{18}$$

$$\gamma_{pq} = \left[1 - (\lambda_{pp}\lambda_{qq} + \lambda_{pq}\lambda_{qp})\right]\sigma_{pq} - \left(\lambda_{pp}\lambda_{pq}\sigma_p^2 + \lambda_{qp}\lambda_{qq}\sigma_q^2\right). \tag{19}$$

Equations (18) and (19) convert the original parameters into error parameters of the VAR process. Besides guaranteeing $\left\{\sigma_p^2, \sigma_q^2\right\}$ to be finite, the VAR coefficients are further restricted so that the computed variances in (16) be non-negative and $\left|\gamma_{pq}\right| / (\gamma_p \gamma_q) \leq 1$.

By looking at the structure specified in (16) and (17), we know which of the two processes are more volatile and more persistent. In addition, we know their correlation and causation relationships. First, it is noted that the VAR structure nests the independence case. Thus we can test to see if the two processes are independent or not. If correlation of the errors and off-diagonal coefficients in the VAR model are statistically small, productivity shock and quality innovation can be considered independent. The test of correlation between the VAR errors is not straightforward because we do not know the variances of the estimated variance-covariance matrix $\widehat{\Sigma}$. It is noted that estimation of the VAR structure brings about unbiased estimates of the VAR coefficients and Σ. Thus we can employ the unbiasedness to simulate many samples and come up with different estimates of Σ and calculate variances of $\widehat{\Sigma}$. Second, the test of causation between quantity and quality can be simply implemented with t-tests on the off-diagonal VAR coefficients.

3 From Ideology to Data

In the basic model, it is straightforward to calculate the quality index. Potentially, there is a mismatch to some extent between the basic model, which is an ideology, and data for two major reasons. First, actual economic contexts do not satisfy all the underlying assumptions of the basic model. Second, observations of relative price, productivity shock, and budget shares are not perfect. Productivity shock and relative price may be imperfectly observed for many reasons, e.g. under-reporting and aggregating over heterogenous and evolving types of commodities. In this section, we discuss what economic contexts the model can be applied to, and consider several ways to deal with imperfect observability and infer quality innovation.

3.1 A Valid Data Set

For an empirical implementation of the basic model, an actual economic context or a data set should possess three critical properties as follows.

Relative Completion. First, the sample should reflect a relatively closed system. To put it differently, variations in relative quantity and price should not be largely influenced by supply and demand outside the economy. If relative completion is violated, relative prices do not bear reliable information on the system's fundamentals, i.e. productivity shock and quality innovation.

Full Equilibrium. Second, variations in nominal prices should fully reflect changes in productivity shock and quality innovation. In equation (7), we see that, relative price, which will be constructed based upon nominal prices, has to adjust to clear commodities markets in equilibrium. If nominal prices are not free to move, relative price does not provide good information on variations deep in the economic system. This also implies that we should not look at high frequency data which potentially have short-run deviations from the fundamentals due to many reasons, e.g. nominal rigidities, unbalanced monetary effects, and speculations. Besides the price-adjustment concern, frequency of data should be low enough for full response of commodity supply and delivery. In other words, data should reflect a system in equilibrium rather than on-going adjustment.

CES Compatibility. Third, the estimated substitution parameter $\widehat{\theta}$ should lie in the interval $(-\infty, 1]$ to be consistent with the CES specification. Recall that $\widehat{\theta}$ is the estimator in a regression, possibly an IV regression, with relative price as the dependent and relative quantity as the independent. If relative quantity and relative price move in opposite directions, $\widehat{\theta}$ is highly likely negative and readily valid. If relative quantity and relative price have positive correlation, the latter should not be too volatile in comparison with the former so that $\widehat{\theta}$ is smaller than unity. In other words, the CES specification is not compatible with too volatile relative price which is positively correlated with productivity shock.

3.2 Matching only with Relative Price

There are several ways to utilize the double manifestation result. One is to retrieve quality innovation only from relative price according to (13), and check how well the model budget share matches with its data counterpart. When applying the basic model to real economic contexts, the inferred quality series may not be totally consistent with the observed budget share. Here are some possible reasons for this potential inconsistency. First, in empirical analyses, the normalized and indexed world only maintains the true growth rates of relative quantity and relative price rather than their true levels. This implies that the computed budget share as defined in (11) does not necessarily match with data counterparts. All we can check is the correlation between them. Second, as mentioned earlier, productivity shock and quality innovation are not perfectly observed. Third, the basic model does not have investment. In reality, this is not the case. Among the three problems mentioned, we will tackle the first and second in the next sections.

Alternatively, we can use budget share data to infer quality and check the result with relative price data. For this, the starting point is (11).

3.3 Double Manifestation and Rescaling

In reality, we often observe productivity shock and relative price as indices. The double manifestation will help us rescale these indices to make model budget share close to its data counterpart. Explicitly, let u and s be the correct rescaling constants. We observe index $(B/A)_t$ for productivity shock and the true productivity shock is $(B/A)_t u$. By the same token, we observe index p_t and rescale it to the true level $p_t s$. Expressions (7) and (11) are rewritten as

$$p_t s = \left(\frac{\beta}{\alpha}\right)_t^{\theta} \left[\left(\frac{B}{A}\right)_t u\right]^{\theta-1}$$

$$S_{at} = \frac{1}{1 + (p_t s)\left[\left(\frac{B}{A}\right)_t u\right]},$$

or

$$p_t us = \left[\left(\frac{\beta}{\alpha}\right)_t u\right]^{\theta} \left(\frac{B}{A}\right)_t^{\theta-1} \tag{20}$$

$$S_{at} = \frac{1}{1 + (p_t us)\left(\frac{B}{A}\right)_t}. \tag{21}$$

In equation (21), it can be seen that the product us can be estimated. However, u and s cannot be individually identified. That also means that we can only infer quality innovation correct up to some unknown scale u, which is used for rescaling productivity shock.

Specifically, the product us is chosen to minimize the squared differences between the model and data budget shares for commodity a as

$$\widehat{us} = \arg\min_x \sum_{t=1}^{T} \left(\left[\frac{1}{S_{at}} - 1\right] - \left[p_t \left(\frac{A}{B}\right)_t\right] x\right)^2. \tag{22}$$

After some simple manipulations, we have the optimal rescaling constant

$$\widehat{us} = \frac{\sum_{t=1}^{T} \left[\frac{1}{S_{at}} - 1\right] \left[p_t \left(\frac{A}{B}\right)_t\right]}{\sum_{t=1}^{T} \left[p_t \left(\frac{A}{B}\right)_t\right]^2}. \tag{23}$$

Next, we rescale p_t with us and estimate θ with an IV estimation. Finally, quality innovation is calculated according to (24).

$$\left(\frac{\beta}{\alpha}\right)_t u = \left[\frac{p_t \widehat{us}}{(B/A)_t^{\theta-1}}\right]^{1/\theta}. \tag{24}$$

Note that estimates for θ are the same for original and rescaled data. Thus, with this rescaling scheme, double manifestation is satisfied by the model to some extent. If we have good level data for relative quantity or relative price, u can be calculated and relative quality will be rescaled to the true level.

3.4 Allowing for Measurement Errors

In the previous section we see that rescaling helps match with the data budget share for a to some extent. In this section, we still use this rescaling scheme and add the unknown factors (u_{1t}, u_{2t}) as in (25) and (26). Expression (25) comes from (20), and equation (26) is derived from (21). For expositional simplicity, we look at the modified budget share rather than the original one. The motivation for these errors is that there are measurement errors in productivity shock, relative price, and budget share. In addition, these are perfectly observed by the agents and not by econometricians. With a multiplicative error structure, imperfect observability is embedded in (u_{1t}, u_{2t}).

$$p_t us = \left[\left(\frac{\beta}{\alpha}\right)_t u\right]^\theta \left(\frac{B}{A}\right)_t^{\theta-1} u_{1t} \tag{25}$$

$$\frac{1}{S_{at}} - 1 = \left[\left(\frac{\beta}{\alpha}\right)_t u\right]^\theta \left(\frac{B}{A}\right)_t^{\theta} u_{2t}. \tag{26}$$

Without multiplicative constants, we do not impose that $E(u_{1t}) = E(u_{2t}) = 1$. However, (u_{1t}, u_{2t}) is assumed to have a finite variance-covariance matrix. It is noted that the scale us is a function of observables as in (23) and θ can be estimated by an IV estimation according to (12).

Given a static world where there are no intertemporal choices, we choose $[(\beta/\alpha)_t u]^\theta$ to minimize the objective function in (27) for any period t

$$\left[\left(\frac{\beta}{\alpha}\right)_t u\right]_{est}^\theta = \arg\min_x \left\{U_t^{'} W U_t\right\} \tag{27}$$

where

$$U_t = \begin{bmatrix} \widetilde{u}_{1t} \\ \widetilde{u}_{2t} \end{bmatrix}, \ \widetilde{u}_{1t} = \frac{1}{u_{1t}} - 1, \ \widetilde{u}_{2t} = \frac{1}{u_{2t}} - 1,$$
$$u_{1t} = \frac{1}{C_{1t}x}, \ C_{1t} = \frac{(B/A)_t^{\theta-1}}{p_t us},$$
$$u_{2t} = \frac{1}{C_{2t}x}, \ C_{2t} = \frac{(B/A)_t^{\theta}}{1/S_{at} - 1},$$

and

$$W = \Omega^{-1}$$
$$\Omega = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$
$$\Omega^{-1} = \frac{1}{Det(\Omega)} \begin{bmatrix} \sigma_2^2 & -\sigma_{12} \\ -\sigma_{12} & \sigma_1^2 \end{bmatrix}$$
$$var(\widetilde{u}_{1t}) = \sigma_1^2, \ var(\widetilde{u}_{2t}) = \sigma_2^2, \ covar(\widetilde{u}_{1t}, \widetilde{u}_{2t}) = \sigma_{12},$$
$$Det(\Omega) = \sigma_1^2\sigma_2^2 - \sigma_{12}^2 = \sigma_1^2\sigma_2^2\left(1 - \frac{\sigma_{12}^2}{\sigma_1^2\sigma_2^2}\right) > 0.$$

It is straightforward to have the minimizer in (27) as (Appendix A.3)

$$\left[\left(\frac{\beta}{\alpha}\right)_t u\right]_{est}^\theta = \frac{\sigma_2^2 C_{1t} + \sigma_1^2 C_{2t} - \sigma_{12}(C_{1t} + C_{2t})}{\sigma_2^2 C_{1t}^2 + \sigma_1^2 C_{2t}^2 - 2\sigma_{12} C_{1t} C_{2t}}, \tag{28}$$

and hence the estimator of relative quality index is

$$\left[\left(\frac{\beta}{\alpha}\right)_t u\right]_{est} = \left[\frac{\sigma_2^2 C_{1t} + \sigma_1^2 C_{2t} - \sigma_{12}\left(C_{1t} + C_{2t}\right)}{\sigma_2^2 C_{1t}^2 + \sigma_1^2 C_{2t}^2 - 2\sigma_{12} C_{1t} C_{2t}}\right]^{1/\theta}. \tag{29}$$

Let q_t be the true relative quality index in period t and let $\Phi\left(\widetilde{u}_{1t}, \widetilde{u}_{2t}; q_t\right)$ be the estimator in (29). The estimator of relative quality index has the following conditional expectation and variance

$$E\left[\Phi\left(\widetilde{u}_{1t}, \widetilde{u}_{2t}; q_t\right)|q_t\right] = q_t E\left\{\left[\frac{\Phi_{Ut}}{\Phi_{Lt}}\right]^{1/\theta}\right\} \tag{30}$$

$$var\left(\Phi\left(\widetilde{u}_{1t}, \widetilde{u}_{2t}; q_t\right)|q_t\right) = \Delta\Phi\left(\widetilde{\mu}_1, \widetilde{\mu}_2; q_t\right)' \Omega \Delta\Phi\left(\widetilde{\mu}_1, \widetilde{\mu}_2; q_t\right), \tag{31}$$

where $\widetilde{\mu}_1 = E\left(\widetilde{u}_{1t}\right)$, $\widetilde{\mu}_2 = E\left(\widetilde{u}_{2t}\right)$, and $\left(\Phi, \Phi_{Ut}, \Phi_{Lt}\right)$ are defined in (A.14) (Appendix A.3). The complex term $E\{[\Phi_{Ut}/\Phi_{Lt}]\}^{1/\theta}$ in (30) is called the *correction factor* whose sample counterpart is defined in (A.17). If the sample correction factor is significantly different from unit, the point estimator and variance in (29) and (31) should be adjusted accordingly.

There are two important notes. First, we do not know Ω at the start. For this reason, the implementation procedure has two steps. In the first step, Ω_1 is the identity matrix. In the second step, Ω_2 is established based upon the estimated errors $\{\widehat{\widetilde{u}}_{1t}, \widehat{\widetilde{u}}_{2t}\}_{t=1}^T$ from the first step (Appendix A.3). In implementation, we can actually repeat the steps until the estimated Ω converges, given a small tolerance level. Second, in the inference procedure, we use two pieces of information to pin down relative quality. This may lead to overidentification. The overidentification test is carried out based on the standard J-statistic which is Chi-squared distributed with one degree of freedom.

The corresponding fitted relative price and budget share are

$$\left(p_t us\right)_{fit} = \left[\left(\frac{\beta}{\alpha}\right)_t u\right]_{est}^{\theta} \left(\frac{B}{A}\right)_t^{\theta-1} \tag{32}$$

$$\left(S_{at}\right)_{fit} = \frac{1}{1 + \left[\left(\frac{\beta}{\alpha}\right)_t u\right]_{est}^{\theta} \left(\frac{B}{A}\right)_t^{\theta}}. \tag{33}$$

As $\left(\widehat{u}_{1t}, \widehat{u}_{2t}\right)$ capture the differences between model outcomes and data counterparts, we define two goodness-of-fit measures

$$\text{relative price} \quad : \quad R_{RPR} = 1 - \left(\frac{1}{T}\sum_{t=1}^{T}\left[\widehat{u}_{1t} - 1\right]^2\right)^{1/2}, \tag{34}$$

$$\text{budget share for } a \quad : \quad R_{BSA} = 1 - \left(\frac{1}{T}\sum_{t=1}^{T}\left[\widehat{u}_{2t} - 1\right]^2\right)^{1/2}. \tag{35}$$

By construction R_{RPR} and R_{BSA} generally live in $[0, 1]$ and tell how much variation in relative price and budget share is explained by productivity shock and quality innovation. In addition, the quantitative role that measurement errors play in a specific context is captured by $(1 - R_{RPR})$ and $(1 - R_{BSA})$.

4 US Services vs. Goods in 1946-2006

In this section, we look at relative productivity shock and quality innovation between two US broad product groups: services and goods, respectively commodities b and a in the theoretical model. The annual data set, which is drawn from the National Income and Product Accounts (NIPA), covers the period 1946-2006 (Appendix A.4).

4.1 Data Description

The data set is valid for the basic model because it satisfies the three critical conditions. First, we can treat the US economy as being relatively closed. Net exports play a small part in total GDP, i.e. 3.2 percent in 1946, -5.8 percent in 2006, and -0.7 percent on average in 1946-2006 (Table A.1). Second, annual data is expected to allow full adjustments in most real activities and nominal prices. Third, we will see that the estimated substitution parameter $\widehat{\theta} \leq 1$, satisfying the CES specification.

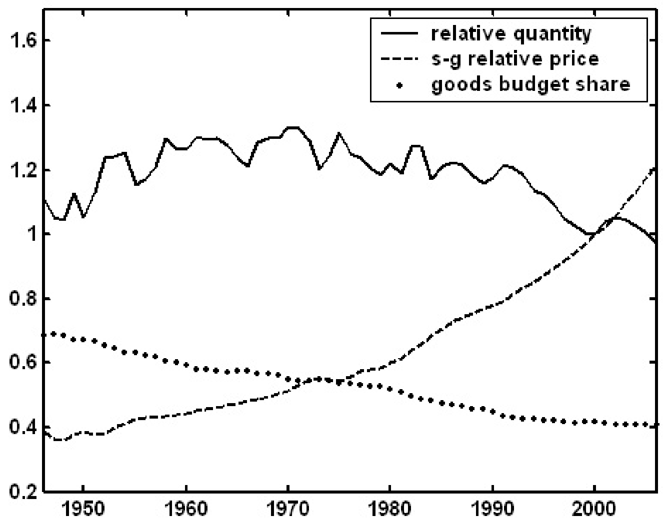
Table 2: Variables in US data set

Description	Definition
Goods quantity index	Q_G
Goods price index	P_G
Services quantity index	Q_S
Services price index	P_S
Budget share for goods*	BSG
US population index*	POP
Services-goods relative quantity*	$SGP = Q_S/Q_G$
Services-goods relative price*	$RPR = P_S/P_G$

Note: (*) unit root at 5%; see Appendix A.4 for more details.

Here are some important details on the construction and use of the variables (Table 2). First, quantity and price indices are constructed with a Fisher's formula, which uses weights from two adjacent years (Appendix A.4). In addition, quantity variables include final sales of domestic product and changes in inventories, and exclude imports. It is noted that we exclude structures in all considerations because they have service flows for an extended period of time, which is hard to be picked up by a static model (Appendix A.4). Second, the budget share for goods is calculated based on private consumption data, which does not include investment, and covers imported goods and services for consumption. It is noted that there is currently no reliable information to separate domestic and imported products in private consumption. As mentioned earlier, we can treat the US economy as relatively closed. Third, US population will be used as the instrumental variable in the estimation of θ. Relative quantity is expected to bear some information about total population. In the mean time, we do not expect a relationship between relative quality and population. Later in the implementation, we will check if total population is a valid instrument.

Figure 1: US data set 1946-2006, year 2000 = 1



Source: constructed from NIPA (BEA).

Time series of relative quantity, relative price, and budget share for goods in 1946-2006 are presented in Figure 1. It can be observed that budget share for goods is decreasing over time. In addition, while relative quantity of services is fluctuating, the relative price has an increasing trend. This latter observation suggests quality innovation may have some effects on the relationship between relative price and productivity shock.

4.2 Quality Information in Data

We have some further notes about the quality information and other noisy information possibly borne by price and budget share data.

First, the current statistical system measures a value index as the product of price and quantity indices, e.g. US BEA's method in (A.18). Let a be physical quantity associated with physical price p. Let αa be efficiency quantity associated with efficiency price $\widehat{p}$. We observe that the value index can be interpreted in different ways, i.e. $p.a = \widehat{p}.\alpha a$. Thus, if we deflate the value index by some price deflator, physical or efficiency, we will have the corresponding quantity index. Conceptually, our quality inference methods rely on the physical price p because it has the quality content. The question is what price data do we currently have, physical price or efficiency price? The answer is a mixture of the two which is closer to physical price. In other words, price data bear information about quality changes to a large extent. It is noted that, the extent to which prices reflect quality is not fixed. There is a gradual evolution from p to $\widehat{p}$ by the moves of different US statistical agencies, especially the Bureau of

Labor Statistics (BLS) whose consumer and producer price indices are used by others. Before 1998, there were quality adjustments to some products like motor vehicles and apparels by the BLS. The Boskin Commission of 1996 reported that price indices are biased upward for not adjusting quality changes. Since 1998, the BLS has used hedonic price regressions more extensively to adjust quality changes in prices. The extent to which prices are adjusted for quality changes is far from complete. Landefeld & Grimm (2000) estimate that, by 2000, 18 percent of US final expenditure is deflated by hedonic prices. Thus the price and quantity data used in the current research are not conceptually perfect as the physical price and quantity, especially after 1998. However, as price data still bear much quality information, the quality inference exercise is valid.

Second, price data do not differentiate between quality improvement and variety growth. Quality improvement means consumers have higher utility from the same quantities of some fixed products. Variety growth means changes in the number of varieties while quality for each variety is constant. Theoretically, variety growth can be equivalently represented by quality improvement, e.g. total utility $\int_0^\theta u(x)\,di$ can be replaced by single utility $\theta u(x)$. Consequently, though explicitly about quality improvement, the basic model can also capture the effects of variety growth if price data bear these effects. In fact, the current statistical practice tends to support this. To see why, we look at an example of two cars of the same model. If they have the same color, each can be sold for ten thousand dollars. If they have different colors which are appreciated by consumers, each can claim eleven thousand dollars. In the second case, though the total quantity is the same, the average price is higher. In practice, the two car variants are recorded in the same category and the average price should bear information on variety growth.

Third, with annual data, we conjecture that the ratio between services and goods prices is not much biased by unbalanced monetary effects. Investigating a large sample in the US consumption price data for 1995-1997, Bils & Klenow (2004) show that it takes a median period of less than six months for prices to change. In addition, the relative frequency of price changes in all goods and services are 26 percent. Specifically, the relative frequencies of price changes for durable goods, nondurable goods, and services are respectively 30, 30, and 21 percent. Even though, the degree of nominal rigidity is not the same for all products, the probability that some price will change after one year is very large. That is, our data frequency is low enough for services and goods prices to bear equivalent monetary effects, and the relative price and budget share mostly capture relative productivity shock and quality innovation.

Fourth, we do not explicitly control production cost. However, production cost is linked to productivity and hence can be summarized by productivity shock. Thus, the basic model already somehow separates the cost effects on relative price and budget share.

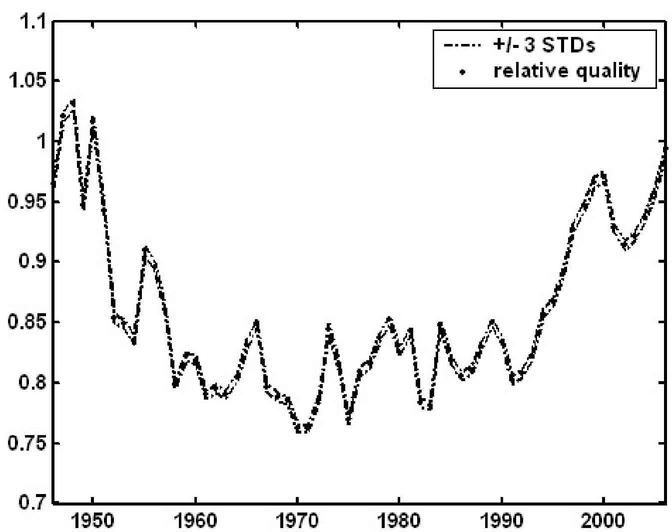
Fifth, we currently do not have information on sales tax to refine price data. However, the tax information remaining in price data may be relatively harmless for several reasons: (i) we are interested in the ratio of two aggregate prices rather than individual price indices; (ii) at the aggregate level, the relative tax rates should be stable for two adjacent years; and moreover (iii) each link, i.e. year-to-year, in the Fisher price index series is not affected by a link far away from that. In other words, effective tax rates do not change much between two adjacent years, and the time series of services-goods relative price should bear noisy tax information only to a small extent relative to productivity and quality effects.

4.3 Services Relative Quality and Parameters

The implementation has three steps: (i) estimating the substitution parameter θ; (ii) inferring the quality index; and (iii) analyzing the dynamic relationship between productivity shock and quality innovation.

There are some notes about validity of the estimates. First, to check the validity of the instrument estimation following (12), we look at the correlation between relative quality and population growth. The correlation is weak at 4 percent. Meanwhile, productivity shock and population index time series are correlated at -43 percent. That means the estimate of θ is reliable. Second, the sample correction factor is found to be unit, i.e. the estimator is unbiased. Third, the J-test rejects the hypothesis of overidentification.

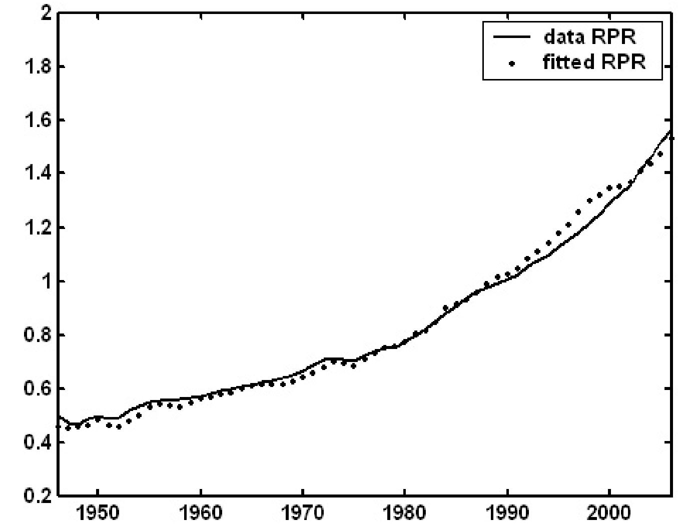
Figure 2: Services-goods relative quality 1946-2006



Note: estimation is based on (29) and (31).

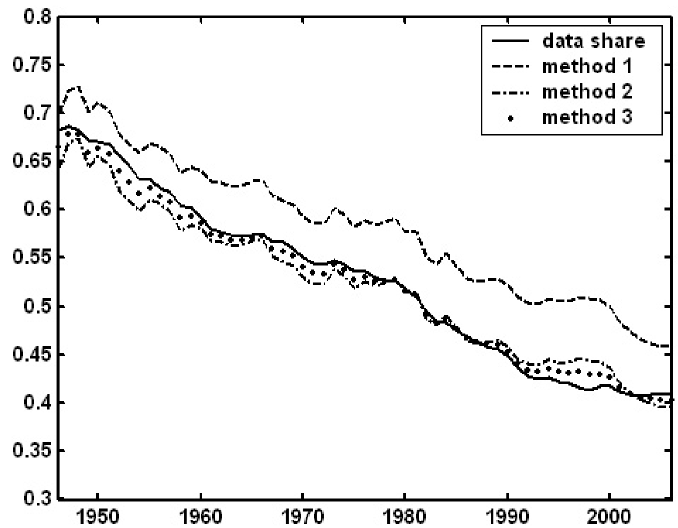
The point estimate for θ is -10, which means a substitution elasticity σ of 0.09 (simple OLS estimate for θ is -2.2, for σ is 0.3). That is goods and services are generally hard to substitute each other. Next, we calculate the quality innovation time series with three methods: (i) matching only with relative price;

Figure 3: Actual and fitted relative prices 1946-2006



Note: fitted RPR in (32).

Figure 4: Actual and fitted budget shares for goods 1946-2006



Note: method 1 in (13); method 2 in (21); method 3 in (33).

(ii) rescaling relative price; (iii) and allowing for measurement errors as discussed in Section 3. The series generated by three methods have very high correlation. The striking result is that relative quality time series following method 2 and method 3 are very close. That means measurement errors play a small role in this specific case. The point estimate and $\pm 3STD$ band for services-goods quality index according to method 3 are presented in Figure 2. It can be seen that services relative quality was decreasing until early 1970s when it started increasing.

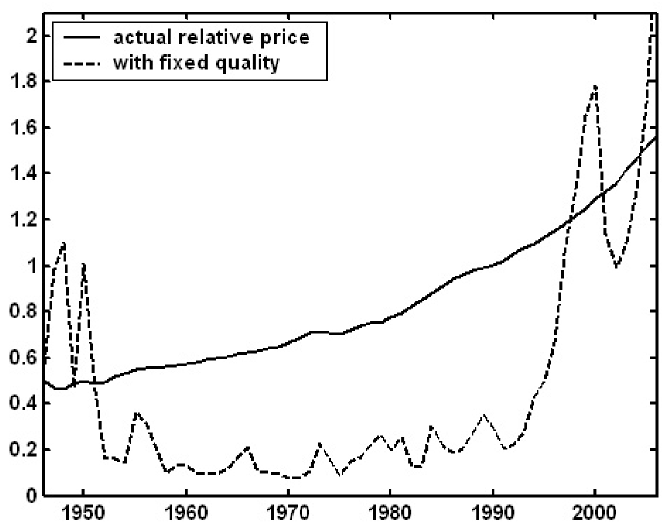
In overall, observed productivity shock and inferred quality innovation help largely explain variations in services-goods relative price and budget share for goods. In fact the measures for goodness of fit are very high, i.e. $R_{RPR} = 0.96$ and $R_{BSA} = 0.97$. Figures 3 and 4 show that the actual and fitted variables are very close to each other.

Table 3: US services-goods: estimation results

Description	Definition	Estimate	STD
	CES specification		
substitution parameter	θ	-10.087	1.238
elasticity of substitution	$\sigma = \frac{1}{1-\theta}$	0.090	
	quantity and quality series		
productivity shock: mean & STD	μ_p & σ_p	1.183	0.098
productivity shock: variation	σ_p/μ_p	0.083	
quality innovation: mean & STD	μ_q & σ_q	0.859	0.071
quality innovation: variation	σ_q/μ_q	0.082	
correlation coefficient	φ	-0.917	
covariance	$\sigma_{pq} = (\sigma_p\sigma_q)\varphi$	-0.006	
	VAR coefficients		
quantity on quantity	λ_{pp}	1.208	0.145
quality on quantity	λ_{qp}	0.425	0.200
quantity on quality	λ_{pq}	-0.182	0.115
quality on quality	λ_{qq}	0.650	0.158
	Σ specification		
variance of quantity error	γ_p^2	0.0020	$1.5e-4$
variance of quality error	γ_q^2	0.0013	$1.3e-6$
error covariance	γ_{pq}	-0.0016	$1.2e-5$
error correlation	$\frac{\gamma_{pq}}{\gamma_p\gamma_q}$	-0.9884	
	Correlation with relative price		
quantity series		-0.627	
quality series		0.281	
	Correlation with instrument–population		
quantity series		-0.430	
quality series		0.044	

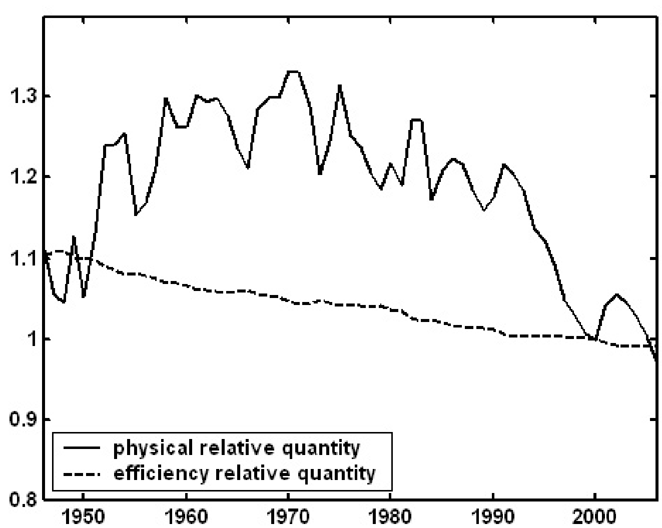
Given both productivity shock and quality innovation time series, we now analyze them by the VAR model discussed earlier. Estimates of the VAR

Figure 5: Actual and counterfactual relative prices 1946-2006



Note: quality index in (32) is kept constant at 0.944.

Figure 6: Physical and efficiency quantities 1946-2006



Note: efficiency quantity equals physical quantity times quality.

structure are presented in Table 3. From the results, we have several observations as follows. First, quality innovation is as volatile as productivity shock ($\sigma_q/\mu_q \approx \sigma_p/\mu_p$). Second, productivity shock is more persistent than quality innovation ($\lambda_{pp} > \lambda_{qq}$). Even though productivity shock is not stationary, we still use it level time series to generate the moments of interest. Third, quality innovation has positive effect on productivity shock while the latter has relatively small negative impact on the former (VAR coefficients). Fourth, productivity shock and quality innovation have negative correlation, which partly comes from large negative correlation between two technical seeds, i.e. error correlation is at -99 percent. This strong result suggests that there is an endogenous trade-off between productivity shock and quality innovation. Fifth, quantity error is more volatile than quality error. Sixth, relative price is correlated with productivity shock at -0.63 and with quality innovation at 0.28 percent. As negatively correlated, productivity shock and quality innovation weaken each other, leading to a less volatile relative price.

4.4 With and without Quality Innovation

To clearly see the role of quality innovation, we carry out two counterfactual analyses, one is on relative price and the other on relative quantity.

First is a counterfactual analysis on the relative price, in which quality index is kept constant and productivity shock alone drives the relative price. Figure 5 shows that productivity shock alone can produce the upward sloping in relative price to some extent. However productivity shock poorly projects the smoothness in relative price. This counterfactual result suggests that if we ignore quality innovation and try to reproduce some relative price, the result can be an estimated quantity series with different properties than reality.

Second is a contrast between physical and efficiency quantities. Figure 6 shows that while physical quantity has a lot of variations, efficiency quantity is much smoother and has a negative trend. Again, this result puts forth a warning on empirical studies: we need the consistency between the objects in consumption, production, and the data counterparts in terms of quality nature. Conditional on questions of interest, an inconsistency between model and data objects may lead to misleading results.

5 Conclusion

The current study develops a model which accounts for variations in both relative quantity and quality between sectors, and potentially between countries. In the model, relative productivity shock and quality innovation are manifested in both relative price and budget share, i.e. double manifestation. In addition, partial effects of productivity shock and quality innovation on relative price and budget share depend on the substitution parameter. The double manifestation result helps separate the unobserved relative quality innovation. Given time series of productivity shock and quality innovation, we can investigate their individual and joint characteristics, i.e. variance, persistency, causation, and correlation.

We then apply the separation method to the services and goods sectors of the US from 1946-2006. The result shows that observed productivity shock and inferred quality innovation explain variations in the relative price and budget share very well. In addition, productivity shock alone fails to explain the smoothness in services-goods relative price. In this specific case, productivity shock and quality innovation are negatively correlated. Essentially, this is a specific case which supports the quality innovation hypothesis, i.e. aggregate quality does change over time.

One possible response to these empirical results is that we have no way to differentiate between quality innovation and taste shock. First, it is noted that the empirical results come from annual data. As already discussed earlier, taste shock is hard to be the case because we do not have a coordination device that can alter all preferences at the same time and in the same direction each year. The hypothesis of quality innovation is more plausible. Quality innovation can be annually achieved via competition, imitation, or any other forms of technology diffusion. In our model, changes in the nature of products will be translated into observable relative prices and budget shares. Second, Nguyen (2007) shows that productivity driven quality (services relative to goods) have a positive trend like that in Figure 2 from 1970-2006. That means we have a good reason to interpret the changes in β/α as quality innovation.

The theoretical and empirical results put forth a warning that growth and business-cycle models should not ignore quality innovation at the start. Specifically, the missing of quality innovation may be relatively harmless in a certain set of growth and business-cycle models. However, by not explicitly modeling quality innovation, models with an emphasis on relative prices may generate misleading results. In a certain context, we need to evaluate the relative importance of quality innovation before simplifying the working model.

Klenow (2003) suggests that "...the BLS begin tabulating the difference between inflation in average unit prices and inflation in the CPI. This gap reflects BLS quality adjustments..." Our study supports this suggestion. More specifically, we should keep both price measures, one for physical price (p), and the other for efficiency price ($\widehat{p} = p/\alpha$). In statistical practice, we are now moving from measuring p towards measuring $\widehat{p}$. Efficiency price $\widehat{p}$ is directly useful for welfare calculations. However, if we do not keep physical price p, we loose the ability to separate quantity and quality. Moreover, if quantity and quality are lumped into one measure, we will have another model of the world. That means we may have some policy descriptions which do not deal with the differences between quantity production and quality innovation, possibly leading to non-optimal outcomes.

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A Appendix

A.1 Equilibrium in the Basic Model

(i) Agent i solves the UMP in period t

$$\max_{\{a_{it},b_{it}\}} \left\{ (\alpha_t a_{it})^{\theta} + (\beta_t b_{it})^{\theta} \right\}^{1/\theta}$$
$$\text{s.t. } a_{it} + p_t b_{it} = e_{ait} + p_t e_{bit}.$$

Equivalently, given the Inada condition, we need to solve

$$\max_{b_{it}>0} \left\{ \alpha_t^{\theta} \left(e_{ait} + p_t e_{bit} - p_t b_{it} \right)^{\theta} + \beta_t^{\theta} b_{it}^{\theta} \right\}^{1/\theta}.$$

The necessary and sufficient condition with respect to b_{it} is

$$\alpha_t^{\theta} \left(e_{ait} + p_t e_{bit} - p_t b_{it} \right)^{\theta-1} p_t = \beta_t^{\theta} b_{it}^{\theta-1}. \tag{A.1}$$

(ii) In equilibrium, we already have that $b_{it} = B_t$. In addition, with the equal-endowment rule, $e_{ait} = A_t$ and $e_{bit} = B_t$. Thus (A.1) can be written as

$$\alpha_t^{\theta} A_t^{\theta-1} p_t = \beta_t^{\theta} B_t^{\theta-1},$$

and the equilibrium relative price is

$$p_t = \left(\frac{\beta_t}{\alpha_t} \right)^{\theta} \left(\frac{B_t}{A_t} \right)^{\theta-1}. \tag{A.2}$$

(iii) Given the equilibrium relative price in (A.2), the equilibrium budget share for commodity a is

$$S_{at} = \frac{1}{1 + p_t \frac{B_t}{A_t}}, \text{ or}$$
$$S_{at} = \frac{1}{1 + \left(\frac{\beta_t}{\alpha_t} \right)^{\theta} \left(\frac{B_t}{A_t} \right)^{\theta}}. \tag{A.3}$$

Figure A.1: Quality innovation and marginal utility, $\theta < 0$

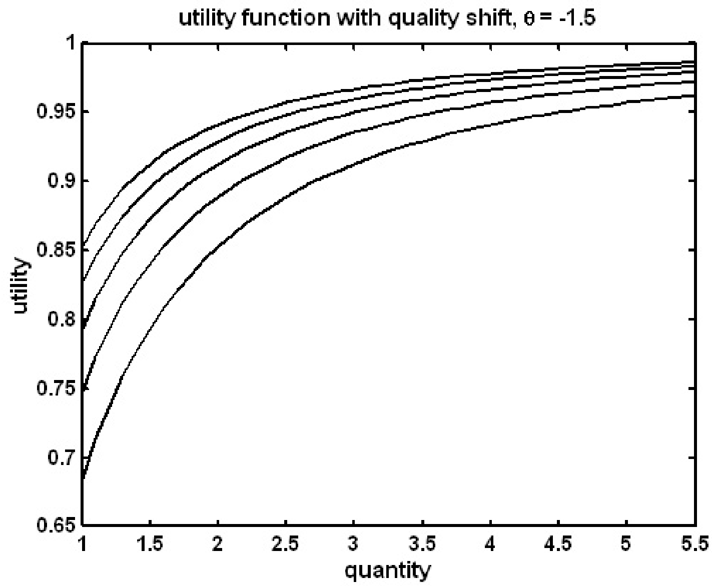
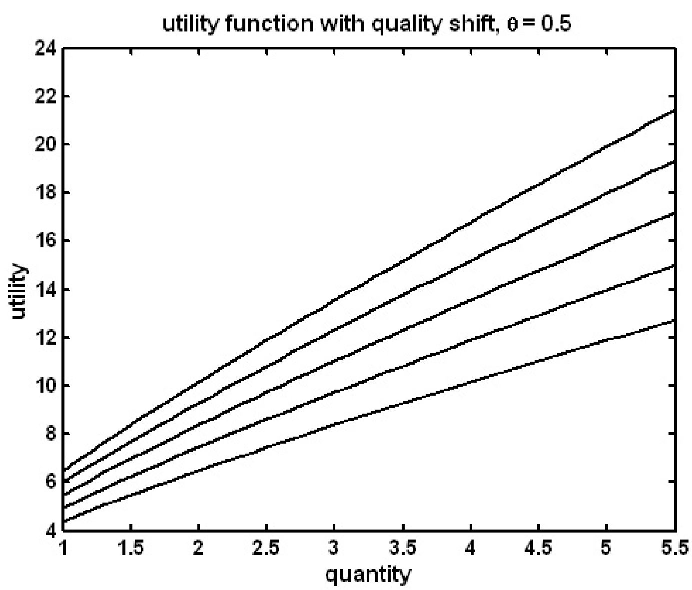


Figure A.2: Quality innovation and marginal utility, $\theta > 0$



A.2 Relations between VAR Parameters

(i) Derivation of variances of $(\varepsilon_{pt}, \varepsilon_{qt})$

From (14) and (15), we have

$$\begin{cases} P_t = \lambda_{pp} P_{t-1} + \lambda_{qp} Q_{t-1} + \varepsilon_{pt} \\ Q_t = \lambda_{pq} P_{t-1} + \lambda_{qq} Q_{t-1} + \varepsilon_{qt} \end{cases} \tag{A.4}$$

$$\Longrightarrow \begin{cases} var(P_t) = var(\lambda_{pp} P_{t-1} + \lambda_{qp} Q_{t-1} + \varepsilon_{pt}) \\ var(Q_t) = var(\lambda_{pq} P_{t-1} + \lambda_{qq} Q_{t-1} + \varepsilon_{qt}) \end{cases}$$

$$\Longrightarrow \begin{cases} \sigma_p^2 = \lambda_{pp}^2 \sigma_p^2 + \lambda_{qp}^2 \sigma_q^2 + \gamma_p^2 + 2\lambda_{pp}\lambda_{qp}\sigma_{pq} \\ \sigma_q^2 = \lambda_{pq}^2 \sigma_p^2 + \lambda_{qq}^2 \sigma_q^2 + \gamma_q^2 + 2\lambda_{pq}\lambda_{qq}\sigma_{pq} \end{cases}$$

$$\Longrightarrow \begin{bmatrix} \gamma_p^2 \\ \gamma_q^2 \end{bmatrix} = \begin{bmatrix} 1-\lambda_{pp}^2 & -\lambda_{qp}^2 \\ -\lambda_{pq}^2 & 1-\lambda_{qq}^2 \end{bmatrix} \begin{bmatrix} \sigma_p^2 \\ \sigma_q^2 \end{bmatrix} - \begin{bmatrix} 2\lambda_{pp}\lambda_{qp}\sigma_{pq} \\ 2\lambda_{pq}\lambda_{qq}\sigma_{pq} \end{bmatrix}. \tag{A.5}$$

(ii) Derivation of covariance of $(\varepsilon_{pt}, \varepsilon_{qt})$

From (A.4) and the fact that $E(P_t) = E(Q_t) = 0$, we have

$$\begin{aligned} covar\left(P_t, Q_t\right) &= \sigma_{pq} = E(P_t Q_t) \\ &= E\left[\left(\lambda_{pp} P_{t-1} + \lambda_{qp} Q_{t-1} + \varepsilon_p\right)\left(\lambda_{pq} P_{t-1} + \lambda_{qq} Q_{t-1} + \varepsilon_q\right)\right] \\ &= \lambda_{pp}\lambda_{pq}\sigma_p^2 + \lambda_{qp}\lambda_{qq}\sigma_q^2 + \left(\lambda_{pp}\lambda_{qq} + \lambda_{pq}\lambda_{qp}\right)\sigma_{pq} + \gamma_{pq}. \end{aligned}$$

Thus

$$\gamma_{pq} = \left[1 - \left(\lambda_{pp}\lambda_{qq} + \lambda_{pq}\lambda_{qp}\right)\right]\sigma_{pq} - \left(\lambda_{pp}\lambda_{pq}\sigma_p^2 + \lambda_{qp}\lambda_{qq}\sigma_q^2\right). \tag{A.6}$$

A.3 Estimation of Relative Quality Index

(i) Deriving relative quality index:

Let $\left[(\beta/\alpha)_t\, u\right]_{est}^{\theta}$ be the modified relative quality index to be estimated in period t. The index is chosen as follows

$$\left[\left(\frac{\beta}{\alpha}\right)_t u\right]_{est}^{\theta} = \arg\min_x \left\{U_t^{'} W U_t\right\} \tag{A.7}$$

where U_t and W are defined in the main text. Let the objective function be

$$F(x) = U_t^{'} W U_t$$

$$F(x) = \frac{1}{Det(\Omega)} \left[\widetilde{u}_{1t}\ \widetilde{u}_{2t}\right] \begin{bmatrix} \sigma_2^2 & -\sigma_{12} \\ -\sigma_{12} & \sigma_1^2 \end{bmatrix} \begin{bmatrix} \widetilde{u}_{1t} \\ \widetilde{u}_{2t} \end{bmatrix}$$

$$F(x) = \frac{\sigma_2^2 \left(C_{1t}x - 1\right)^2 + \sigma_1^2 \left(C_{2t}x - 1\right)^2 - 2\sigma_{12}\left(C_{1t}x - 1\right)\left(C_{2t}x - 1\right)}{Det(\Omega)}.$$

The FOC and also SOC is

$$F^{'}(x) = 0$$

$$\Longrightarrow \left[\sigma_2^2 C_{1t}^2 + \sigma_1^2 C_{2t}^2 - 2\sigma_{12} C_{1t} C_{2t}\right] x = \left[\sigma_2^2 C_{1t} + \sigma_1^2 C_{2t} - \sigma_{12}\left(C_{1t} + C_{2t}\right)\right]$$

Finally, the estimated modified index is

$$x = \frac{\sigma_2^2 C_{1t} + \sigma_1^2 C_{2t} - \sigma_{12}\left(C_{1t} + C_{2t}\right)}{\sigma_2^2 C_{1t}^2 + \sigma_1^2 C_{2t}^2 - 2\sigma_{12} C_{1t} C_{2t}}. \tag{A.8}$$

(ii) The two-step procedure

In the first step, the variance-covariance matrix is

$$\Omega_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the corresponding solution is

$$x = \frac{C_{1t} + C_{2t}}{C_{1t}^2 + C_{2t}^2}. \tag{A.9}$$

The estimated errors are

$$\widehat{\widetilde{u}}_{1t} = \frac{C_{1t}C_{2t} - C_{2t}^2}{C_{1t}^2 + C_{2t}^2}, \tag{A.10}$$

$$\widehat{\widetilde{u}}_{2t} = \frac{C_{1t}C_{2t} - C_{1t}^2}{C_{1t}^2 + C_{2t}^2}. \tag{A.11}$$

Let $e_{1t} = \widehat{\widetilde{u}}_{1t} - \left(\sum \widehat{\widetilde{u}}_{1t}\right)/T$ and $e_{2t} = \widehat{\widetilde{u}}_{2t} - \left(\sum \widehat{\widetilde{u}}_{2t}\right)/T$, we have $\widehat{\Omega} = E^{'}E$, where

$$E = \begin{bmatrix} e_{11} & e_{21} \\ .. & .. \\ e_{1t} & e_{2t} \\ .. & .. \\ e_{1T} & e_{2T} \end{bmatrix},$$

and use $\widehat{\Omega}$ for the second step estimation.

(iii) Conditional expectation and variance of estimated quality index

Let q_t be a true quality index (up to some unknown scale) and $(\widetilde{u}_{1t}, \widetilde{u}_{2t})$ be defined in (27), the estimated quality index based on (A.8) can be rewritten as

$$\Phi\left(\widetilde{u}_{1t}, \widetilde{u}_{2t}; q_t\right) = q_t \left[\frac{\Phi_{Ut}}{\Phi_{Lt}}\right]^{1/\theta} \tag{A.12}$$

where

$$\begin{aligned} \Phi_{Ut} &= \sigma_2^2\left(\widetilde{u}_{1t} + 1\right) + \sigma_1^2\left(\widetilde{u}_{2t} + 1\right) - \sigma_{12}\left(\widetilde{u}_{1t} + \widetilde{u}_{2t} + 2\right), \\ \Phi_{Lt} &= \sigma_2^2\left(\widetilde{u}_{1t} + 1\right)^2 + \sigma_1^2\left(\widetilde{u}_{2t} + 1\right)^2 - 2\sigma_{12}\left(\widetilde{u}_{1t} + 1\right)\left(\widetilde{u}_{2t} + 1\right). \end{aligned}$$

The estimated index has the following conditional expectation

$$E\left[\Phi\left(\widetilde{u}_{1t}, \widetilde{u}_{2t}; q_t\right)|q_t\right] = q_t E\left\{\left[\frac{\Phi_{Ut}}{\Phi_{Lt}}\right]^{1/\theta}\right\}. \tag{A.13}$$

Let $\widetilde{\mu}_1 = E(\widetilde{u}_{1t})$ and $\widetilde{\mu}_2 = E(\widetilde{u}_{2t})$. By the Delta method with reference to the means, conditional variance of the estimated quality index is

$$var(\Phi|q_t) = \Delta\Phi(\widetilde{\mu}_1, \widetilde{\mu}_2; q_t)' \, \Omega \Delta\Phi(\widetilde{\mu}_1, \widetilde{\mu}_2; q_t) \tag{A.14}$$

where

$$\Delta\Phi(\widetilde{\mu}_1, \widetilde{\mu}_2; q_t) = \begin{bmatrix} \partial\Phi(\widetilde{u}_{1t}, \widetilde{u}_{2t}; q_t) / \partial\widetilde{u}_{1t} \\ \\ \partial\Phi(\widetilde{u}_{1t}, \widetilde{u}_{2t}; q_t) / \partial\widetilde{u}_{2t} \end{bmatrix}_{(\widetilde{\mu}_1, \widetilde{\mu}_2)},$$

specifically

$$\begin{aligned}
\frac{\partial\Phi}{\partial\widetilde{u}_{1t}} &= D_t \left[\frac{\left(\sigma_2^2 - \sigma_{12}\right)\Phi_{Lt} - 2\left[\sigma_2^2(\widetilde{u}_{1t}+1) - \sigma_{12}(\widetilde{u}_{2t}+1)\right]\Phi_{Ut}}{\Phi_{Lt}^2} \right], \\
\frac{\partial\Phi}{\partial\widetilde{u}_{2t}} &= D_t \left[\frac{\left(\sigma_1^2 - \sigma_{12}\right)\Phi_{Lt} - 2\left[\sigma_1^2(\widetilde{u}_{2t}+1) - \sigma_{12}(\widetilde{u}_{1t}+1)\right]\Phi_{Ut}}{\Phi_{Lt}^2} \right], \\
D_t &= \frac{q_t}{\theta}\left[\frac{\Phi_{Ut}}{\Phi_{Lt}}\right]^{1/\theta - 1}.
\end{aligned}$$

In empirical studies, the correction factor $E\left[(\Phi_{Ut}/\Phi_{Lt})^{1/\theta}\right]$ in (A.13) is estimated by

$$E\left\{\left[\frac{\Phi_{Ut}}{\Phi_{Lt}}\right]^{1/\theta}\right\}_{est} = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{\Phi_{Ut}}{\Phi_{Lt}}\right]^{1/\widehat{\theta}}, \tag{A.15}$$

where $(\widetilde{u}_{1t}, \widetilde{u}_{2t})$ are derived from the multiplicative residuals in the second-step estimation. If the estimated correction factor is significantly different from unit, the point and variance estimates of the estimated relative quality index should be adjusted accordingly.

A.4 US Services-Goods Data Set

The annual data set on US services and goods covers the period 1946-2006. The series are mainly retrieved from NIPA tables which are reported by the Bureau of Economic Analysis. The series on population is from the estimates of the US Census Bureau. Classifications of goods and services follow the definitions of NIPA tables. The broad components of goods industries are agriculture, forestry, and fisheries; mining; and manufacturing. The services industries are transportation and public utilities; wholesale trade; retail trade and automobile services; finance, insurance, and real estate; different services; and government services.

The original data set has the following variables: (1) US population index (US Census); (2) goods quantity index (NIPA 1.2.3); (3) goods price index (NIPA 1.2.4); (4) services quantity index (NIPA 1.2.3); (5) services price index (NIPA 1.2.4); and (6) budget share for goods (NIPA 1.5.5). It is noted that we leave residential and non-residential structures out of the data set. Some data features are worth noted as follows.

First, goods are both durable and nondurable. we rely on quantity flows of new durable goods rather than service flows from durable stocks. The reason for not using services flows is that stocks of durable goods are composed of different quality levels which are unknown. The same reason applies to the omission of residential and non-residential structures, which can render services for a very long period of time.

Second, the bottom line of the current NIPA tables is that: "...Percent changes in real GDP and its components are equal to the percent changes of the quantity indexes; percent changes in prices are equal to the percent changes of the price indexes..." (A Guide to the NIPA's by the BEA, 2001). Technically, chain-type quantity and price indices are based on Fisher (F) formula which uses weights from two adjacent years, i.e. a combination of Laspeyres (L) and Paasche (P) indices. Specifically, let q's and p's be quantities and prices, Fisher quantity index of period t relative to that of period $t-1$ is

$$Q_t^F = \sqrt{Q_t^L \times Q_t^P}, \tag{A.16}$$

where

$$\begin{aligned} Q_t^L &= \frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}} \\ Q_t^P &= \frac{\sum p_t q_t}{\sum p_t q_{t-1}}; \end{aligned}$$

and by the same token, Fisher price index of period t is

$$P_t^F = \sqrt{P_t^L \times P_t^P}, \tag{A.17}$$

where

$$\begin{aligned} P_t^L &= \frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}} \\ P_t^P &= \frac{\sum p_t q_t}{\sum p_{t-1} q_t}. \end{aligned}$$

Correspondingly, the value index is defined as

$$V_t = P_t^F . Q_t^F . \tag{A.18}$$

The intuition behind (A.16) and (A.17) is that if quantities or prices do not change, $Q_t^F = 1$ or $P_t^F = 1$, respectively. To put it differently, (A.16) reflects only changes in aggregate quantity, and (A.17) is only for variations in aggregate price. The product $Q_t^F \times P_t^F$ is the growth rate of the nominal value between time t and time $t-1$ (A.18). Based on this observation, in practice, most GDP components' nominal values and price indices are derived first from different

Federal Government surveys. Then, starting with the most detailed level for which all the necessary data are available, nominal values are deflated to have real values or quantities (NIPA Help, BEA website).

Third, the construction of year-to-year quantity and price indices are based on the set of commodities existing in two adjacent years. If the set of varieties not shared between two adjacent years is relatively small, which is highly likely the case, time series of aggregate quantity and aggregate price are reliable for the quality inference procedure.

Besides the variables which will be used in the separation exercise, we look at the composition of GDP from the expenditure perspective to see how much the United States depends on the Rest of the World. Table A.1 shows that we can treat the US as relatively closed.

Table A.1: Relative completion of the US economy 1946-2006, percent

Accounts	1946	2006	1946-2006
Gross domestic product	**100.0**	**100.0**	**100.0**
Personal consumption expenditures	**64.9**	**70.0**	**64.5**
Goods	44.3	28.6	33.8
Services	20.6	41.4	30.7
Gross private domestic investment	**14.0**	**16.7**	**16.0**
Goods	7.2	7.8	7.6
Structures	6.8	8.9	8.4
Net exports of goods and services	**3.2**	**−5.7**	**−0.7**
Exports	6.4	11.1	7.5
Goods	5.3	7.8	5.6
Services	1.1	3.3	1.9
Imports	3.2	16.8	8.2
Goods	2.3	14.2	6.6
Services	0.9	2.6	1.6
Government expenditures & investment	**17.8**	**19.0**	**20.2**

Source: Table 1.5.5, NIPA, US Bureau of Economic Analysis.